

2.6 Trig functions and limits

2014-09-25 day 22

There is a semi-important technique to
know in 2.6.

Remember: $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

So ... $\lim_{2x \rightarrow 0} \frac{\sin 2x}{2x} = 1$ and $\lim_{3x \rightarrow 0} \frac{\sin 3x}{3x} = 1$ and so on

but when is $2x$ headed to?

$$2x \rightarrow 0 \text{ iff } x \rightarrow 0$$

So ... $\lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = 1$, $\lim_{x \rightarrow 0} \frac{\sin 3x}{3x} = 1$, and so on...

2.6 Trig functions and limits

2014-09-25 day 22

how do we use these facts?

$$\lim_{x \rightarrow 0} \frac{2 \sin(2x)}{2x} = 2 \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x} = 2(1)$$

$$= 2(1) = 2$$

$$\left. \begin{aligned} \lim_{x \rightarrow 0} cf(x) \\ = c \lim_{x \rightarrow 0} f(x) \end{aligned} \right\}$$

Caveat: x is in RADIANS

2.6 Trig functions and limits

2014-09-25 day 22

$$y = \frac{\sin(x)}{x} \quad \text{red arrow from } x \text{ to } 1$$

$$y = \frac{\sin(2x)}{x} \quad \text{red arrow from } x \text{ to } 2$$

$$y = \frac{\sin(3x)}{2x} \quad \text{red arrow from } 2x \text{ to } \frac{3}{2}$$

$$" \div (2x) "$$

2.6 Trig functions and limits

2014-09-25 day 22

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin(3x)}{2x} &= \frac{1}{2} \lim_{x \rightarrow 0} \frac{\sin(3x)}{x} \\ &= \frac{1}{2} \lim_{x \rightarrow 0} \frac{3 \sin(3x)}{3x} = \frac{3}{2} \lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} = \frac{3}{2}\end{aligned}$$

2.6 Trig functions and limits

2014-09-25 day 22

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{\sin(2x)}{3x} &= \lim_{x \rightarrow 0} \left(\frac{2}{3} \right) \frac{\sin(2x)}{2x} = \frac{2}{3} \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x} \\
 &= \frac{2}{3} (1) = \frac{2}{3}
 \end{aligned}$$

$\lim_{x \rightarrow a} c f(x) = c \lim_{x \rightarrow a} f(x)$
 (Note: In the original image, the constants 2 and 3 in the fraction $\frac{2}{3}$ are circled in red, and the limit of $\frac{\sin(2x)}{2x}$ is indicated as 1 with a bracket and the number 1 written below it.)

2.6 Trig functions and limits

2014-09-25 day 22

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(7x)} = \lim_{x \rightarrow 0} \frac{3}{7} \frac{\sin(3x)}{\sin(7x)}$$

$$= \frac{3}{7} \lim_{x \rightarrow 0} \left(\frac{7x}{\sin(7x)} \right) \left(\frac{\sin(3x)}{3x} \right)$$

$$= \frac{3}{7} (1)(1) = \frac{3}{7}$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin x}$$

$$= \frac{1}{\lim_{x \rightarrow 0} \frac{\sin x}{x}}$$

$$= \frac{1}{(1)} = 1$$

$$\frac{\sin(3x)}{(3x)} = \sin \quad \lim_{x \rightarrow 0}$$

2.6 Trig functions and limits

2014-09-25 day 22

$$\lim_{x \rightarrow 0} \frac{\tan(4x)}{\sin(7x)} = \lim_{x \rightarrow 0} \frac{\left(\frac{4}{7}\right) \frac{\sin(4x)}{\cos(4x)} \sin(7x)}{\sin(7x)} = \frac{4}{7}$$

$$\lim_{x \rightarrow 0} \frac{\left(\frac{\sin(4x)}{\cos(4x)}\right) \left(\frac{1}{\sin(7x)}\right)}{\left(\frac{\sin(7x)}{1}\right) \left(\frac{1}{\sin(7x)}\right)} = \lim_{x \rightarrow 0} \frac{\frac{\sin(4x)}{\cos(4x) \sin(7x)}}{\frac{\sin(7x)}{\sin(7x)}}$$

$$\frac{\left(\frac{a}{b}\right) \left(\frac{1}{c}\right)}{\frac{c}{1} \left(\frac{1}{c}\right)} = \frac{a}{bc} = \lim_{x \rightarrow 0} \frac{\sin(4x)}{\cos(4x) \sin(7x)}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{a}{1} \cdot \frac{c}{b} = \frac{ac}{b}$$

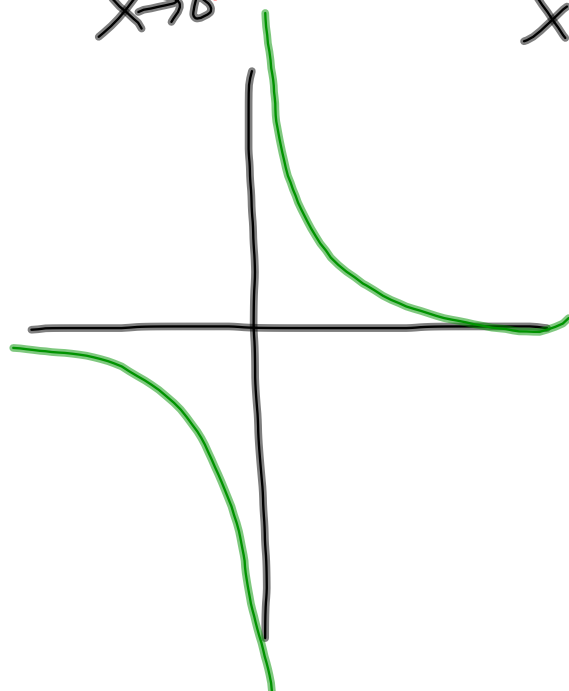
$$\lim_{x \rightarrow 0} \frac{\sin(4x)}{\cos(4x) \sin(7x)} = \frac{4}{7} \lim_{x \rightarrow 0} \frac{\sin(4x)}{4x} \cdot \frac{1}{\cos(4x)} \cdot \frac{7x}{\sin(7x)}$$

$$\lim_{x \rightarrow 0} \left(\frac{4}{7}\right) \left(\frac{\sin(4x)}{4x}\right) \left(\frac{1}{\cos(4x)}\right) \left(\frac{7x}{\sin(7x)}\right)$$

2.6 Trig functions and limits

2014-09-25 day 22

18) $\lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} = \lim_{x \rightarrow 0^+} \left(\frac{\sin x}{x} \right) \left(\frac{1}{x} \right) = +\infty$



$\rightarrow 1$
 $\rightarrow +\infty$
 $-\infty$
DUE

2.6 Trig functions and limits

2014-09-25 day 22

$$\begin{aligned}
 2) \lim_{x \rightarrow 0^+} \frac{\sin(x)}{5\sqrt{x}} &= \lim_{x \rightarrow 0^+} \left(\frac{x}{5\sqrt{x}} \right) \left(\frac{\sin x}{x} \right) \\
 &\rightarrow \lim_{x \rightarrow 0^+} \left(\frac{\sqrt{x}}{5} \right) \left(\frac{\sin x}{x} \right) = 0 \\
 &\quad \downarrow \quad \downarrow \\
 &\lim_{x \rightarrow 0} \left(\frac{\sqrt{x}}{5} \right) \cdot \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)
 \end{aligned}$$

On the right side, a vertical sequence of terms is written in blue:

$$\frac{x^1}{x^{1/2}} = x^{1-1/2} = x^{1/2} = \sqrt{x}$$